

Neuroscience and the Teaching of Mathematics

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Abstract

Much of the neuroimaging research has focused on how mathematical operations are performed. Although this body of research has provided insight for the refinement of pedagogy, there are very few neuroimaging studies on how mathematical operations should be taught. In this article, we describe the teaching of algebra in Singapore schools and the imperatives that led us to develop two neuroimaging studies that examined questions of curricular concerns. One of the challenges was to condense issues from classrooms into tasks suitable for neuroimaging studies. Another challenge, not particular to the neuroimaging method, was to draw suitable inferences from the findings and translate them into pedagogical practices. We describe our efforts and outline some continuing challenges.

Keywords: neuroscience, neuroimaging, algebra, mathematics teaching, Singapore, model method

One distinction that is seldom mentioned in the neuroscience literature on mathematics is the difference between doing mathematics versus teaching or learning mathematics. The former is about how mathematical facts and processes are used to solve mathematical problems. This includes, for example, how primary or elementary school students perform arithmetic computation and quantity estimation. Examples from the secondary school years include how older students perform calculus, trigonometry, and algebraic questions. Although research on the teaching or learning of mathematics may also be concerned about arithmetic or algebraic computation, the focus is on the acquisition of such knowledge and how best to teach them.

Much of the available work in the neurosciences has focused on doing mathematics. The works of Dehaene and his colleagues (for a review, see Dehaene *et al.*, 2003), for example, examined the neuroanatomical systems responsible for the processing of different mathematical operations (addition, subtraction) and processes (exact calculation versus estimation). Their findings suggested that individuals represent numbers on a mental number line. Furthermore, mental arithmetic—subtraction and division, in particular—activate the intraparietal sulci, which is involved in quantitative processes involving the number line. These findings are important and tell us how people process mathematical information. However, they provide no direct evaluation of how one should go about teaching arithmetic or encourage the development of number representation.

A point we want to emphasise is the importance of extending such research to the study of pedagogy. Take, for example, the teaching of mental subtraction. One activity often used by teachers is closely related to mental number lines. Imagine if young pupils are asked how many sweets are left when three sweets are taken away from five. Such questions can be solved by counting or direct retrieval when numbers are small. It is much more difficult when larger numbers are involved and algorithms have to be used, e.g. taking away 3 sweets from 500. One alternative, especially for weaker pupils, is to ask them to imagine a number line and counting backwards from 500. In order to use such mental number lines, the activity is first introduced with smaller numbers. Students then continue to construct and grow the number line. Such activities often have positive pedagogical effects. However, does such facilitation result from the development of students' number lines, children finding such activities fun and engaging, or some other process? Questions such as these are important, as they will help define the efficacious boundaries of specific pedagogies. As discussed in the following sections, such questions are beginning to be addressed in the neuroscience literature.

The Neuroscience of Pedagogy

Though it is important to ascertain the neuroanatomical substrate of specific mathematical processes, it is equally important to examine how such processes are acquired. In the past, the task of translating findings from the laboratory to the classroom has largely been left to educators. As shown by recent works conducted by Delazer (Delazer *et al.*, 2005) and in our laboratory, neuroscience can play a significant role in pedagogical research. Delazer and her colleagues (2005), for example, investigated the cortical correlates of learning-by-strategy versus learning-by-rote. Participants were trained using one or the other method for over a week. The results showed that learning-by-rote activated the left angular gyrus, possibly reflecting the language dependent nature of the strategy. Learning-by-strategy, on the other hand, activated the precuneus, which the authors attributed to the use of visual imagery.

In our laboratory, we have conducted two studies investigating heuristics commonly used to teach algebraic word problems: the model method and symbolic algebra (Lee *et al.*, 2007). Problem solving is the core of the Singapore mathematics curriculum. Primary students (9 to 11-year-olds) are taught to use the model method to solve algebra word problems that, in many countries, are taught later in the curriculum using symbolic or formal algebra. The model method is a diagrammatic representation of a given problem (Ng & Lee, 2005; Ng & Lee, 2009). Take for example a question in which students are told, 'a cow weighs 150 kg more than a dog; a goat weighs 130 kg less than the cow; altogether the three animals weigh 410 kg'. Students are then asked to find the weight of the cow. Using the model method, students use rectangles to represent the unknown values as well as the quantitative relations presented in word problems (see Figure 1). Students find the weight of the cow by undoing the arithmetic procedures. It is not until students are in secondary school (12-year-olds +) when they are taught to solve such problems by constructing a system of equivalent algebraic linear equations.

Although the original intention in using the model method was to give students earlier access to complex word problems (Kho, 1987), access to non-algebraic heuristics

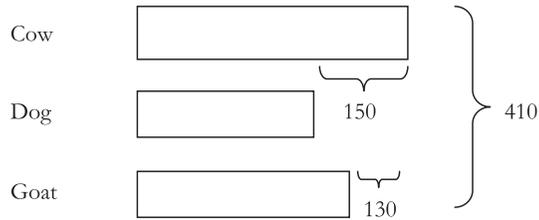


Figure 1: A model representation of an algebraic problem

may also complicate the acquisition of symbolic algebra. For some algebraic word problems, knowledge of such heuristics allows students to arrive at the correct solution without having to engage with the representational and transformational activities associated with symbolic algebra. Some students use a mixed method strategy: combining the heuristic approach with aspects of symbolic algebra. Others construct a model drawing and then its equivalent algebraic equation before reverting to the arithmetic methods of undoing the operations. Indeed, Khng and Lee (2009) found students with poorer inhibitory abilities had difficulties using symbolic algebra even when they were directed to do so. Such students were also more likely to revert to the model method.

The potential pitfalls of introducing pre-algebraic methods are echoed by some secondary teachers' beliefs regarding the model method. Ng, Lee, Ang, and Khng (2006) reported interviews with several secondary teachers regarding the efficacy of the model method. Many of these teachers expressed reservations about the method and saw it as an obstacle to students' acquisition of symbolic algebra. One of their concerns is that the model method is childish and is non-algebraic.

How accurate are these teachers' perceptions regarding differences between the two methods? If the model method is indeed non-algebraic and poses an obstacle to the learning of symbolic algebra, it may be advisable to forego the teaching of the model method. A full evaluation of this issue will likely require a consideration of cognitive, motivational, and pedagogical issues. Because the model method is taught in all schools and has been so for over a decade, conventional programme evaluation techniques are impracticable.

In our laboratory, we conducted two studies using functional magnetic resonance imaging (fMRI) and focused on the cognitive underpinnings of the two methods. In the first study (Lee *et al.*, 2007), we asked adult participants to transform word problems into either models or equations. One of the challenges was to transform problems used in the classroom to a format suitable for use in a scanner. Classroom problems, even those used in primary schools, can be complex and require several minutes to solve. Children typically have access to pen and paper, which serve as external mnemonic aids. In some classrooms, problems are solved in a collaborative manner. Because the acquisition of images requires participants to be still and silent, it is difficult to provide such support materials in the scanner. Perhaps because of this, previous studies that examined algebraic problems have tended to focus only on numerical transformation.

Our aim was to examine differences resulting from the use of the model versus the symbolic method in solving word problems. For this reason, too narrow a focus on a specific subset of processes would not have captured the gist of our enquiry. To achieve

a balance between internal and external validity, we used very simple word problems, e.g. 'John has 34 more cup cakes than Mary. How many cup cakes does John have?' To isolate processes involved in transforming word problems into models versus equations, total quantity was not presented. This discouraged participants from engaging in computation.

All participants in our study were pretested for competency in the two methods: we selected only those who were highly and similarly competent. Ensuring behavioural equivalence allowed us to infer differences in neural activation in terms of processes involved in executing the two methods rather than differences in task difficulty. Despite the lack of behavioural differences, we found differences in the degree to which the two methods activated areas associated with attentional and working memory processes. In particular, transforming word problems into algebraic representation required greater access to attentional processes than did transformation into models. Furthermore, symbolic algebra activated the caudate, which has been associated with activation of proceduralised information (Anderson *et al.*, 2004).

In a follow-up study, we investigated the next stage in problem solving: from models or equations to solution. Using models and equations derived from questions similar to those in the first study, participants were asked to solve the problems and to come up with a solution. Preliminary analyses revealed a similar pattern of findings with symbolic algebra activating areas associated with working memory and attentional processes. Findings from these two studies suggest that, for simple algebraic questions at least, differences between the methods are quantitative rather than qualitative in nature. Both methods activate similar brain areas, but symbolic algebra imposes more demands on attentional resources.

Pedagogical Implications

If symbolic algebra is indeed more demanding on attentional resources, one curricular implication is that it is best to teach the model method at the primary level and leave symbolic algebra until students are more cognitively matured. In evaluating this recommendation, it is important to note that participants in our neuroimaging studies were adults and were all similarly proficient in the two methods. Our recommendation assumes that similar differences will be found in younger learners; indeed, using symbolic algebra may require even more effort for early learners of algebra. Nonetheless, these are empirical assumptions that require further investigation.

We mentioned earlier that some secondary school students adopt a mixed-method strategy in solving algebra problems. In the following section, we discuss the intervention used to address this issue. Of relevance is that the neuroimaging findings provided some insights on why the intervention had limited success.

Students' use of a mixed method approach is of concern to educators because knowledge of symbolic algebra is critical for solving problems in higher mathematics and in the sciences. To address this issue, the Singapore Ministry of Education and the National Institute of Education jointly developed '*Algebar*', a software tool designed to (a) help students make the link between the model method and symbolic algebra and (b) acquire the direct algebraic route of problem solution. Given an algebra word problem,

students who would otherwise use a mixed method approach are prompted by the software to construct equivalent algebraic equations, beginning with definitions of the variables. The inbuilt self-checking system provides instant feedback that supports the construction of algebraic equations.

In late 2006, this software was piloted in two secondary schools, involving four teachers. Participants (~13 year olds) were first taught the symbolic manipulation and transformational activities related to symbolic algebra. Working in pairs, they used *Algebar* to solve a set of algebra word problems. We videotaped the interactions between eight pairs of students. The dyadic interactions showed that many times, students drew an appropriate model representation, constructed a set of equations, and were then unsure how to proceed with constructing a set of equivalent equations that would lead them to the solution.

Focusing on the teachers, analysis of these lessons showed that they used a transmission paradigm to deliver the content. Rather than explaining the procedures necessary for constructing a system of equivalent linear equations, students were *told* how to transform equations. In the post-lesson interviews, teachers explained that their pedagogy was constrained by the limited curricular time allocated to the teaching of symbolic manipulation and transformational activities.

Why do students find constructing a system of equivalent equations difficult? One possible reason is that the pedagogy used to teach students symbolic manipulation and transformational activities was not meaningful. Because the time spent on introducing algebra was brief, students may not have sufficient time to master the procedures. This is particularly problematic if we consider our imaging findings. They show that even for simple algebra problems, symbolic algebra is resource intensive relative to the model method. One solution is to spend more time on introductory activities associated with symbolic algebra. If the procedures related to the construction of a system of equivalent linear equations are explained and are then rehearsed until they are automatized, students may find it easier to adopt the symbolic route to the solution of problems.

As part of this effort, the second author of this article offers professional development courses to help teachers enhance their pedagogy. The objectives of these courses are to inform teachers how they can use *Algebar* to make the link between the two methods and to help improve the teaching of symbolic manipulation and transformational activities. One specific objective is to deploy strategies that will help students reduce the working memory demands of symbolic algebra.

Conclusions

In this article, we focused on two issues. The first issue is related to the distinction between doing versus teaching mathematics: knowing how specific mathematical processes are implemented will not necessarily tell us how best to teach them. Second, one of the challenges in drawing useful information from the neurosciences is to bridge the divide between the laboratory and the classroom. The suggestion that neuroscience may inform the work of mathematics educators often elicits raised eyebrows from colleagues, who respond by asking whether it involves installing a fMRI machine in schools and scanning children to determine the state of their brains. More seriously, replicating the

group-based characteristics of pedagogy in a platform designed for individual investigation is non-trivial. Although the efficacy of pedagogy can be investigated on a one-to-one basis, what works in an individual setting may not be effective in a classroom setting. A closely related concern relates to the context in which learning occurs. What works in a controlled laboratory environment may not do so in a classroom environment. Research in education has also emphasised the importance of discourse amongst community of learners (Brown & Campione, 1994); again something that is difficult to implement within the confines of a scanner.

Although these observations may portray a negative view of future progress, this is not our intention. Some of the identified problems are technical in nature. Recent developments in near-infrared spectroscopy and electroencephalography promise both portability and higher tolerance for movement, both of which may allow for more naturalistic examination of pedagogical strategies.

Note

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